Technical Note

1965-15

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A Shaped Reflector as a Primary Feed for Haystack

30 September 1965

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Group 61

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ABSTRACT

This paper describes a technique for obtaining high aperture
efficiency in the Haystack 120-foot diameter Cassegrain antenna
system. The approach used seeks to maximize the percentage of
power intercepted by the hyperboloidal subreflector of the
Cassegrain antenna system by shaping its primary feed radiation
pattern. The primary feed configuration is a linearly polarized
waveguide feed illuminating a specially shaped reflector. A number
of theoretical antenna models were calculated and one of these
antennas (a 30 wavelength diameter shaped reflector at 7750 Mcps)
was fabricated and evaluated. Calculated estimates of percentage of
primary feed power intercepted by the subreflector and Cassegrain
aperture efficiency are presented.

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A Shaped Reflector as a Primary Feed for Haystack

INTRODUCTION

An effort is made to maximize the percentage of power intercepted by the subreflector of a Cassegrain antenna system. The technique employed seeks to improve the beam efficiency of the primary feed by shaping its radiated pattern in such a way as to illuminate the hyperboloidal subreflector with a pattern whose chief characteristics are a flat top and steeply sloped sides. The accompanying phase is also calculated at a distance R_o which represents the separation between the primary feed and the subreflector.

This paper will describe the method used to design a feed with the characteristics stated above. A number of theoretical antenna models are calculated; one of these antennas was built and its measured data compared with the theory. A technique for correcting the primary feed phase deviation is also described. In addition, calculated estimates of percentage of primary feed power intercepted by the subreflector and the Cassegrain aperture efficiency are presented.

I.   DESCRIPTION OF PRIMARY FEED

A series of Clavin[1,2] waveguide feeds, linearly polarized, used for illuminating paraboloids have been built as primary feeds for the Haystack Cassegrain antenna system. Although these feeds are compact and versatile, such as providing 1.0 Gcps bandwidth, low cross polarized energy and continuous rotation of polarization capability; the beam efficiency defined as that percentage of primary feed power intercepted by the subreflector measures 70 percent. A well designed horn[3,4] can be constructed to allow the

subreflector to intercept approximately 90 percent of the incident power.

In an effort to approach this efficiency and still maintain the afore-mentioned advantages of the Clavin feed it was considered feasible to shape a reflecting surface to the Clavin feed radiation in order to obtain a highly efficient flat-topped secondary radiation pattern.

A paraboloid aperture diameter of ten wavelengths was used to illuminate Haystack's subreflector with an average 12 db edge illumination. It was realized that in order to maintain sufficient gain after reshaping a reflector, the aperture diameter would need to be greatly increased. Thirty wavelengths was arbitrarily chosen as a new reflector diameter. The focal length of the feed was set such that the shaped reflector subtends an angle of 180° at the feed to obtain a maximum interception of incident power by the reflector.

The expression for determining the shape of a line to be used in designing the reflector configuration is found as follows.[5]
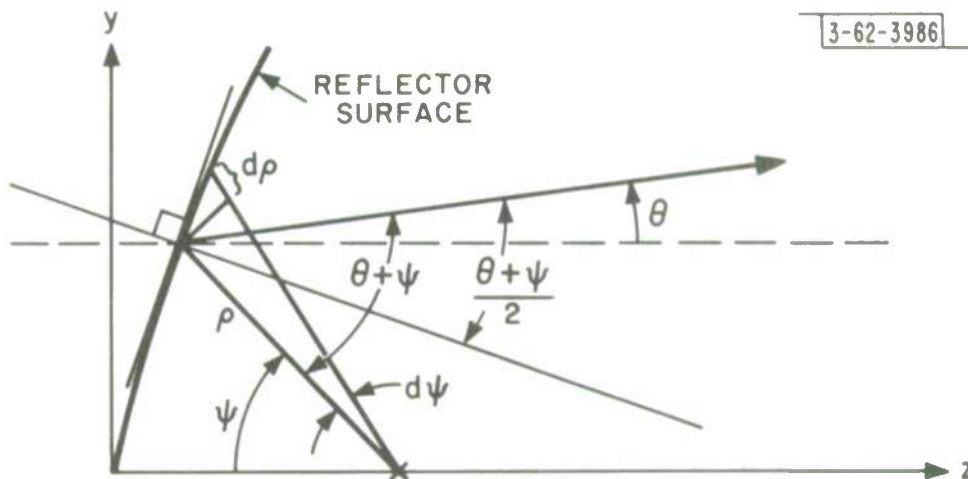
Fig. 1. Ray Geometry at reflector surface.

The relationships of the angles ψ and θ and the curve shape is seen from the figure to be

$$\frac{d\rho}{\rho d\psi} = \tan \frac{\psi + \theta}{2} \tag{1}$$

and the integral is

$$\ell_n \frac{\rho}{\rho_o} = \int_o^\psi \tan \left[\frac{\psi + \theta(\psi)}{2} \right] d\psi \tag{2}$$

The angle $\theta(\psi)$ must be determined before the above expression can be used.

Fig. 2.   Ray geometry for determining $\theta(\psi)$.

Referring to Fig. 2 and utilizing the energy balance principle of geometric optics the power within an angular segment defined by ψ and ψ_1 is equal to the power within the angular segment defined by θ and θ_1. As it is desired

to include all of the power within an angular segment defined by θ_2 and θ_1 the following relationship is established

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{\int_{\psi_1}^{\psi} I(\psi)\,d\psi}{\int_{\psi_1}^{\psi_2} I(\psi)\,d\psi}$$

$I(\psi)$ is the illumination power function of the primary feed.

(3)

setting $\theta_1 = 0$

$\psi_1 = 0$

ψ_2 = angle at edge of reflector

$$\theta(\psi) = \theta_2 \frac{\int_{o}^{\psi} I(\psi)\,d\psi}{\int_{o}^{\psi_2} I(\psi)\,d\psi}$$

(4)

It is the parameter θ_2 which is used to determine $\theta(\psi)$.

As an initial approximation, the angle $\theta_2 = 6.7^\circ$ was chosen because it was desired that all of the power in the secondary pattern be included within a cone whose half angle is 6.7°. The illumination function used in the calculation is the power distribution of the Clavin feed. With the $\theta(\psi)$ calculation completed it is now possible to insert these values into the equation

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for $\ell_n \rho/\rho_o$ and compute the line shape. When this line is rotated about the
focal axis through 360° the surface of the specially shaped reflector is
generated.

II. CALCULATION OF CURVED REFLECTOR RADIATION PATTERN

The scattered field E will be calculated starting with the expression
derived in Silver[6]

$$E = \int_{S_o} \frac{[G_f(\theta,\phi)]^{\frac{1}{2}}}{\rho} \, [\eta \times (\overline{\rho} \times \overline{e}_1)] \, e^{-jk[e-(\overline{\rho}\cdot\overline{R}_1)]} \, ds. \tag{5}$$

$\overline{e}_1$ is a unit vector defining the polarization of the electric field intensity.
The polarization will be assumed constant and will be neglected. The expres-
sion reduces to

$$E = \int_{S_o} \frac{[G_f(\theta,\phi)]^{\frac{1}{2}}}{\rho} \, e^{-jk[\rho-(\overline{\rho}\cdot\overline{R})]} \, ds \tag{6}$$

To evaluate the phase term in the near field the distance R_o and R_1 to a
point P in the near field must be considered. Referring to Fig. 3

$$R_1^2 = [\rho \sin\phi \, \sin\psi]^2 + [\rho \cos\phi + R_o \cos\theta]^2 + [R_o \sin\theta - \rho \sin\phi \, \cos\psi]^2. \tag{7}$$

The total distance to point P from phase reference point O by way of point A
on the reflector is $\rho + R_1$

$$\rho + R_1 = \rho + R_o \left[1 + 2 \frac{\rho}{R_o} (\cos\phi \, \cos\theta - \sin\theta \, \sin\phi \, \cos\psi) + \left(\frac{\rho}{R_o}\right)^2 \right]^{\frac{1}{2}} \tag{8a}$$

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and the phase difference is:

$$\rho + R_1 - R_0 = \rho + R_0 \left[1 + 2 \frac{\rho}{R_0} (\cos\phi \cos\theta - \sin\theta \sin\phi \cos\psi) \right.$$

$$\left. + (\frac{\rho}{R_0})^2 \right]^{\frac{1}{2}} - R_0 \tag{8b}$$

which can be simplified to

$$\rho + R_1 - R_0 = \rho \left[1 + \cos\theta \cos\phi - \sin\theta \sin\phi \cos\psi + \frac{\rho}{2R_0} \right] . \tag{8c}$$
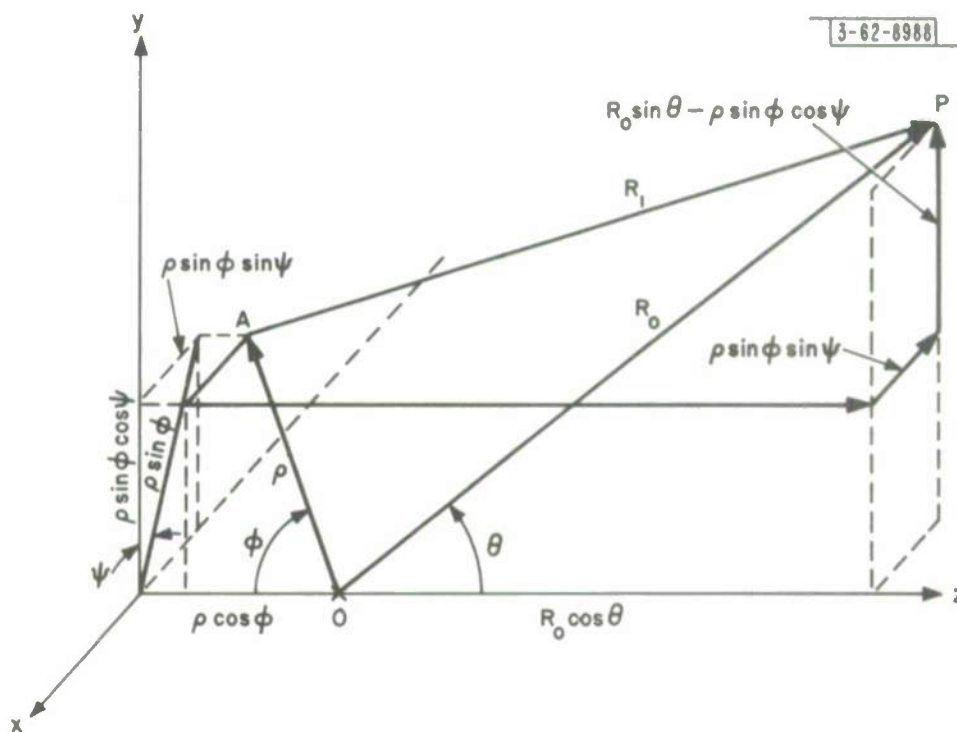

Fig. 3.   Geometry used for determining near field phase expression.

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The expression for the field is:

$$E = \int_{S_o} \frac{[G_f(\theta,\phi)]^{\frac{1}{2}}}{\rho} e^{-jk\rho[1+\cos\theta \cos\phi-\sin\theta \sin\phi \cos\psi+\frac{\rho}{2R_o}]} ds. \qquad (9)$$

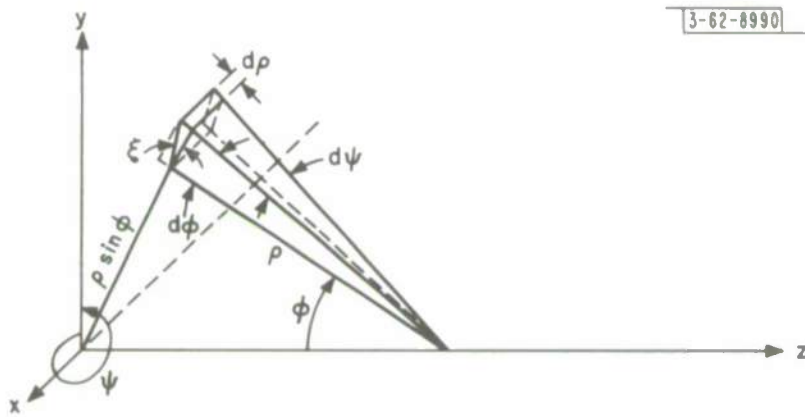To evaluate ds, the elemental area, refer to Fig. 4.

Fig. 4.   Elemental projection on reflector surface.

The projection on the reflector surface is ds

$$ds = [(\rho d\phi)^2 + (d\rho)^2]^{\frac{1}{2}} \rho \sin\phi \ d\psi \qquad (10)$$

$$ds = \rho d\phi \{1 + (\frac{d\rho}{\rho d\phi})^2\}^{\frac{1}{2}} \rho \sin\phi \ d\psi \qquad (11)$$

$$= [1 + \tan^2\xi]^{\frac{1}{2}} dA \qquad (12)$$

$$= \sec\xi \ dA. \qquad (13)$$

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$$ds = \sec\xi \; \rho^2 \; \sin\phi \; d\phi d\psi \tag{14}$$

rewriting the equation and noting that the radiation pattern is invariant with ψ.

$$E = \int_0^{\phi_0} \int_0^{2\pi} \frac{[G_f(\phi)]^{\frac{1}{2}}}{\rho(\phi)} \; \rho^2 \; \sec\xi \; \sin\phi$$

$$\cdot \; e^{-jk\rho(\phi)[1+\cos\theta \; \cos\phi - \sin\theta \; \sin\phi \; \cos\psi + \frac{\rho}{2R_0}]} \; d\phi d\psi \tag{15}$$

and for a circular aperture

$$E = \int_0^{\phi} [G_f(\phi)]^{\frac{1}{2}} \; \rho(\phi) \; [1 + (\frac{d\rho}{\rho d\phi})^2]^{\frac{1}{2}} \; \sin\phi \; J_0(k\rho \sin\theta \sin\phi)$$

$$\cdot \; e^{-jk\rho[1+\cos\phi \; \cos\theta + \frac{\rho}{2R_0}]} \; d\phi. \tag{16}$$

Phase Correction:— The phase front of a spherical wave at the subreflec-
tor is a locus of constant phase described by a radius R_0. The Cassegrain
antenna system transforms the primary spherical wave illumination over the
subreflector into a spherical wave illumination over the paraboloid which
appears to originate from the common paraboloid and hyperboloidal focii. In
this way the reflector system produces a collimated wave front whose rays are
parallel to the focal axis. The shaped reflector phase front is not spheri-
cal. Figure 5 and Fig. 6 consider two different shaped reflectors and plot

the phase distribution over the main reflector as a function of its radius r.
A similar phase distribution which could be produced by an axial displacement,
δ_o, of the apparent feed point is also shown. Combining these two distri-
butions it is possible to obtain a nearly spherical wave illumination of the
main reflector, as shown in Figs. 5 and 6. This could be accomplished by
moving the subreflector along the focal axis. It is seen in Fig. 5 that a
half-wave displacement could result in a maximum resultant phase error of
0.11λ at the edge of the paraboloid. Figure 5 indicates the phase relation-
ship for the parameter θ_2 in Eq. (4) equal to 4.5^o. Figure 6 indicates the
same for $\theta_2 = 3.5^o$.

Results of Theoretical Calculations:— The significant parameters for
consideration are the percentage of power intercepted by the subreflector and
the resultant Cassegrain aperture efficiency. Calculations were performed to
determine this percentage of power for various shape reflectors.

The expression for gain and the method used for calculating the beam
efficiency is as follows. Assuming circular symmetry about the axis of the
beam, the gain of the feed, G, is given by:

$$G = \frac{4\pi}{2\pi \int_o^\pi E^2 \sin\theta d\theta}$$

where E is the field illumination obtained from Eq. (16) and the fraction of
power, ζ, intercepted by the hyperboloid is given by

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$$\zeta = \frac{\int_{o}^{6.7^{o}} E^2 \sin\theta d\theta}{\int_{o}^{\pi} E^2 \sin\theta d\theta}$$

where 6.7^{o} is one-half the angle subtended by the subreflector at the feed.

$$\zeta = \frac{G}{2} \int_{o}^{6.7^{o}} E^2 \sin\theta d\theta.$$

Figure 7 shows plots of beam efficiency as percent power vs. angle θ. Of interest is the percentage of power within 6.7^{o} which is half the angle subtended by the subreflector at the primary feed. The first approximation of $\theta_2 = 6.7^{o}$ is the poorest at 50 percent power. However, it is seen that curve shapes of $\theta_2 = 4.5^{o}$ and less indicate greater than 80 percent beam efficiency. In fact, the $\theta_2 = 3.0^{o}$ curve indicates that 92 percent of the power would be intercepted by the subreflector. It will be assumed that the remaining 8 percent is lost in the side lobes and in the small cross polarized component (-30 db).

Also calculated was the aperture efficiency[7] η which would result from these various shapes by again assuming a circular beam cross section and correct phase.

$$\eta = 16 \left(\frac{f}{D}\right)^2 M^2 G_f(o) \left| \int_{o}^{6.7^{o}} E(\theta) \tan\frac{\theta}{2} d\theta \right|^2$$

where

$G_f(o)$ = on-axis gain of primary feed

M = magnification factor of Haystack geometry

$\frac{f}{D}$ = focal length to diameter ratio of Haystack paraboloid

$E(\theta)$ = voltage distribution of primary feed

θ = primary feed angle taken from focal axis.

Table 1 is a list of results showing the percentage of power intercepted by the subreflector, the on-axis gain of the shaped reflector and the resulting Cassegrain aperture efficiency calculated.

TABLE 1

	$\theta_2 = 3.0^\circ$	3.5°	4.0°	4.5°	5.0°	6.7°
percent power intercepted by subreflector	92	89	85.5	81	75	50.5
η aperture efficiency of Cassegrain antenna	87	88	86	83	76	51
G(o) db shaped reflector on-axis gain	24.29	24.84	25.28	25.3	24.84	19.92

Figure 8 is a plot showing a family of radiation patterns resulting from a change in the parameter θ_2. The $\theta_2 = 4.5^\circ$ curve illuminates the subreflector edge at -4.0 db; however, 81 percent of the power is intercepted by the hyperboloid. The $\theta_2 = 3.5^\circ$ radiation pattern illuminates the subreflector edge 6.7 db down from peak and the percentage power intercepted by the subreflector is 89 percent.

III. MEASUREMENTS

A 30 wavelength diameter shaped reflector design of $\theta_2 = 4.5^O$ was con-
structed and evaluated. Antenna patterns and gain measurements were obtained
with the conditions simulating the near field relationship of the primary
feed and subreflector separation in the Cassegrain antenna system. Results
of these measurements are compared with the calculated radiation pattern and
are shown in Fig. 9. The principal plane and 45° plane patterns which exhibit
good symmetry were averaged and the result is shown as a single illumination
function in Fig. 10. Figure 11 compares the calculated beam efficiency with
the test antenna measured beam efficiency obtained with the measured gain and
illumination function. The shaped reflector design frequency was 7750 Mcps.

The following table lists these results.

TABLE 2

	Theoretical	Measured Test Antenna
percent power to hyperboloid	80.5	78.5
η Cassegrain aperture efficiency	82.8	81.9
G(o) db shaped reflector on-axis gain	25.3	25.0

CONCLUSION

The 120-foot Haystack Cassegrain antenna aperture efficiency obtained
with a series of Clavin-feed paraboloids has been determined to be 60 percent.
This aperture efficiency does not take into account losses due to (1) aperture

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block, (2) errors in the surface of the surface of the subreflector and the 120-foot paraboloid and (3) the 150-foot radome which is appreciable ($\approx$ 1.2 db at 8.0 Gcps).

A technique has been described for improving this antenna aperture efficiency by reshaping the beam of the primary feed and repositioning the subreflector along the focal axis. Measurements on one shaped reflector antenna agreed closely with the theory. Listed below is a table comparing possible solutions.

Reflector Shape θ_2	Percent Power Intercepted by Subreflector	Subreflector Edge Illumination Referred to Maximum Illumination	Calculated Aperture Efficiency
3.0°	92 percent	-9.2 db	87 percent
3.5°	89	-6.7	88
4.0°	85.5	-4.8	86
4.5°	81	-4.0	83

Reflector shapes $\theta = 3.0^\circ$, 3.5° offer the best solutions and would appear to promise the highest Cassegrain antenna aperture efficiency obtainable utilizing the technique described in this paper.

ACKNOWLEDGMENTS

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7. A. Dion, "The Aperture Efficiency of Some Horn-fed Paraboloidal Reflectors," Group Report 61G-3, Lincoln Laboratory, M.I.T. (30 December 1963).
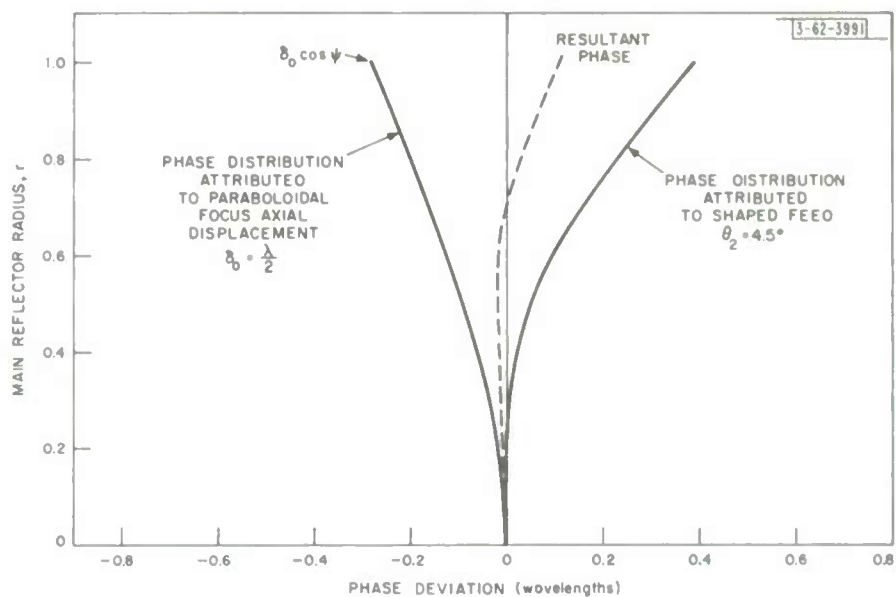
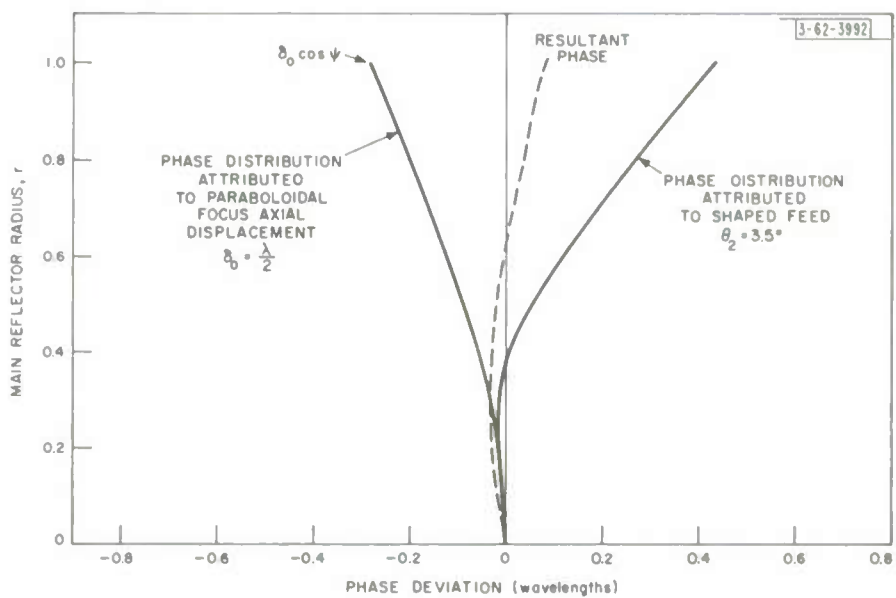Fig. 5.   Phase distribution to 120-foot paraboloid, $\theta_2 = 4.5^{\circ}$.

Fig. 6.   Phase distribution to 120-foot paraboloid, $\theta_2 = 3.5^{\circ}$.
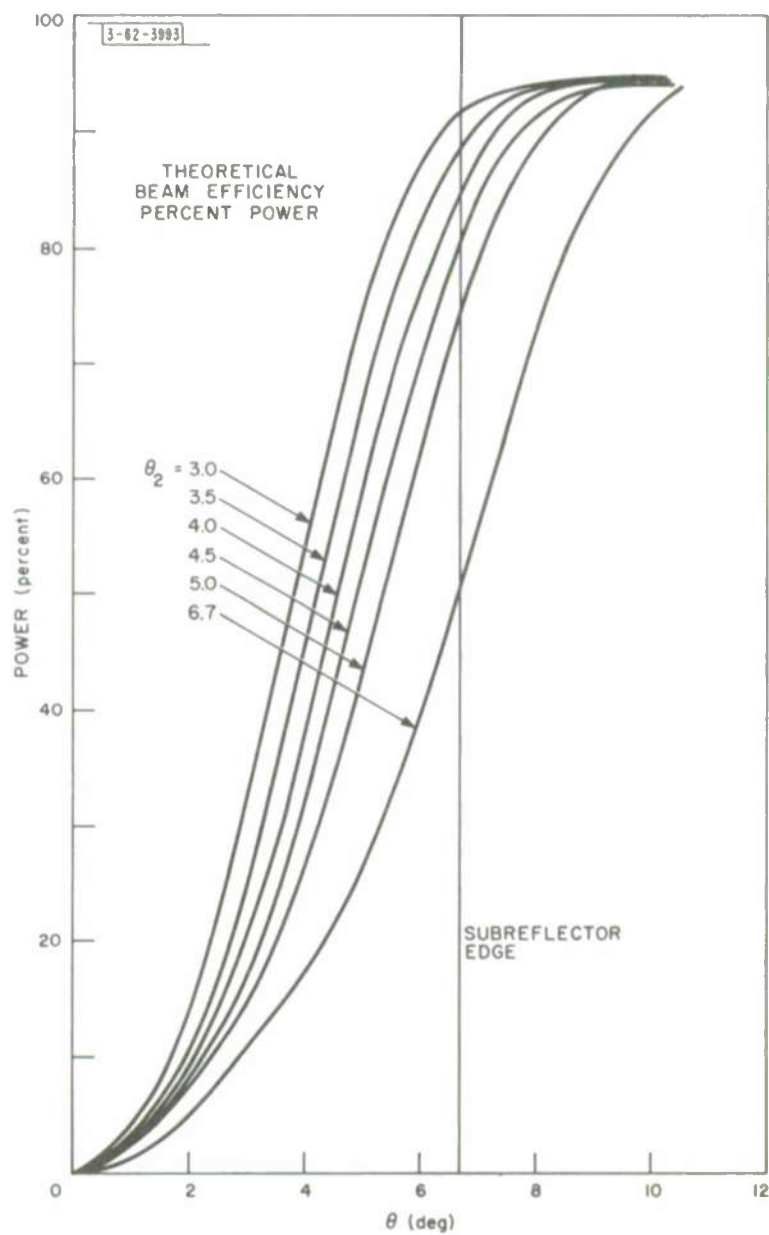
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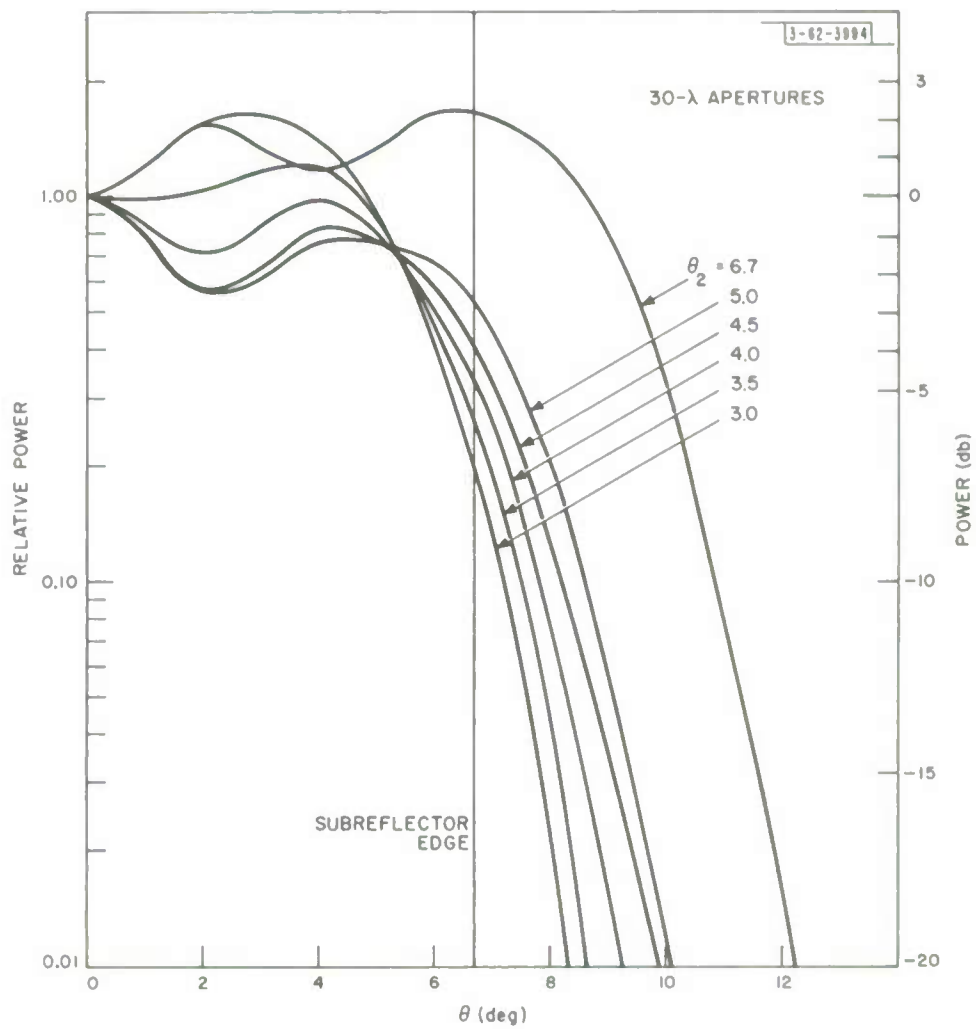Fig. 7.   Calculated beam efficiency of a number of shaped reflectors.

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Fig. 8. Calculated antenna pattern for a number of shaped reflectors.
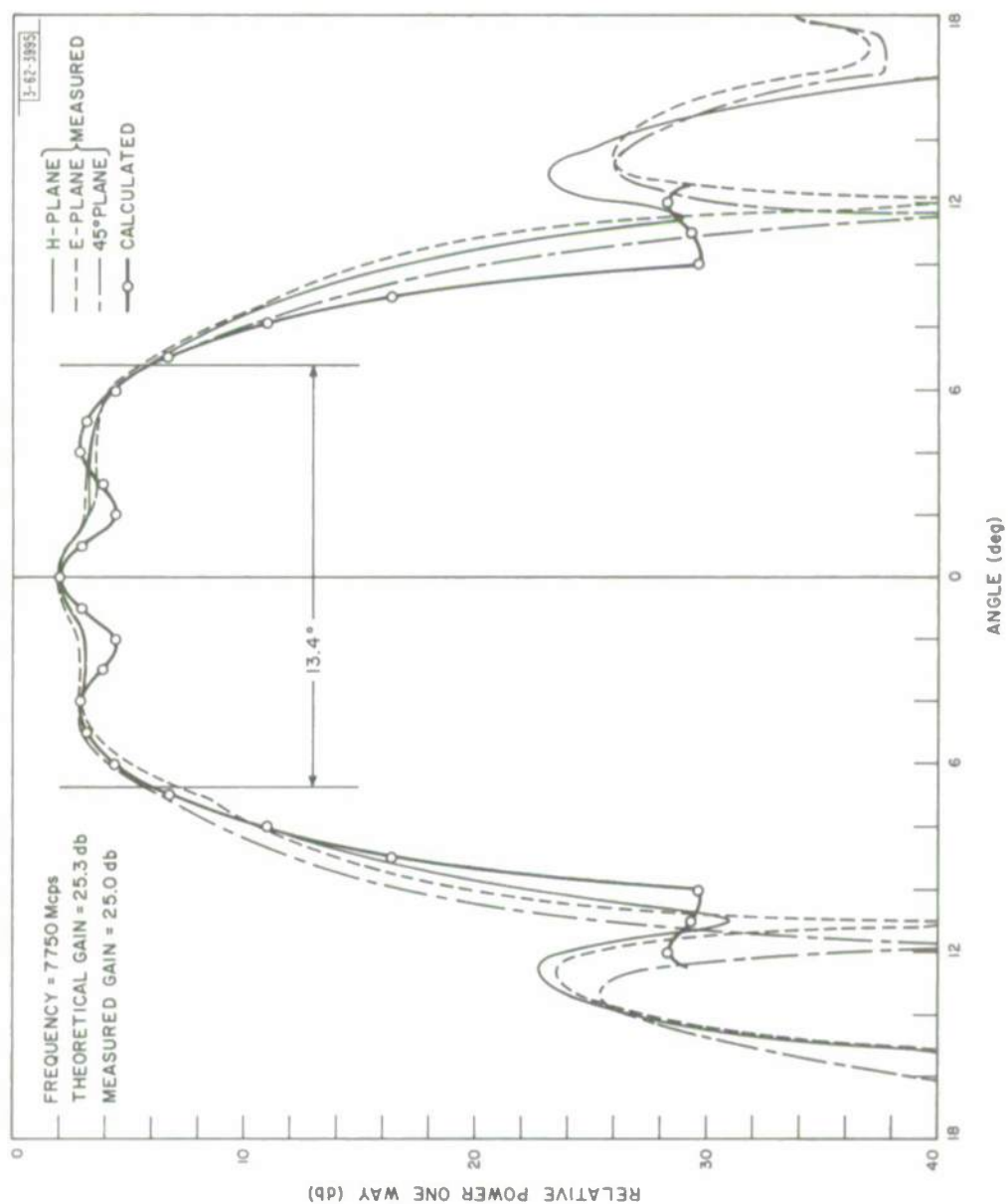
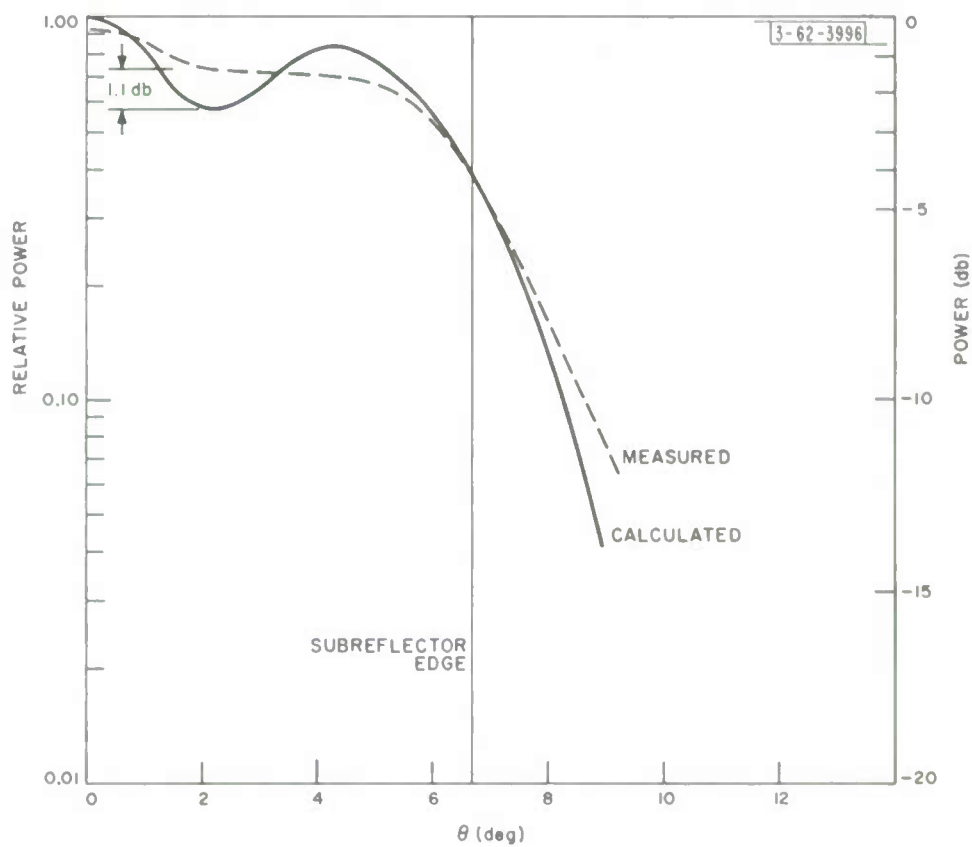Fig. 9. Comparison between calculated and measured $\Theta_2 = 4.5^{\circ}$ shaped reflector.

Fig. 10. Comparison between calculated and average of measured patterns of shaped reflector ($\theta_2 = 4.5^\circ$).
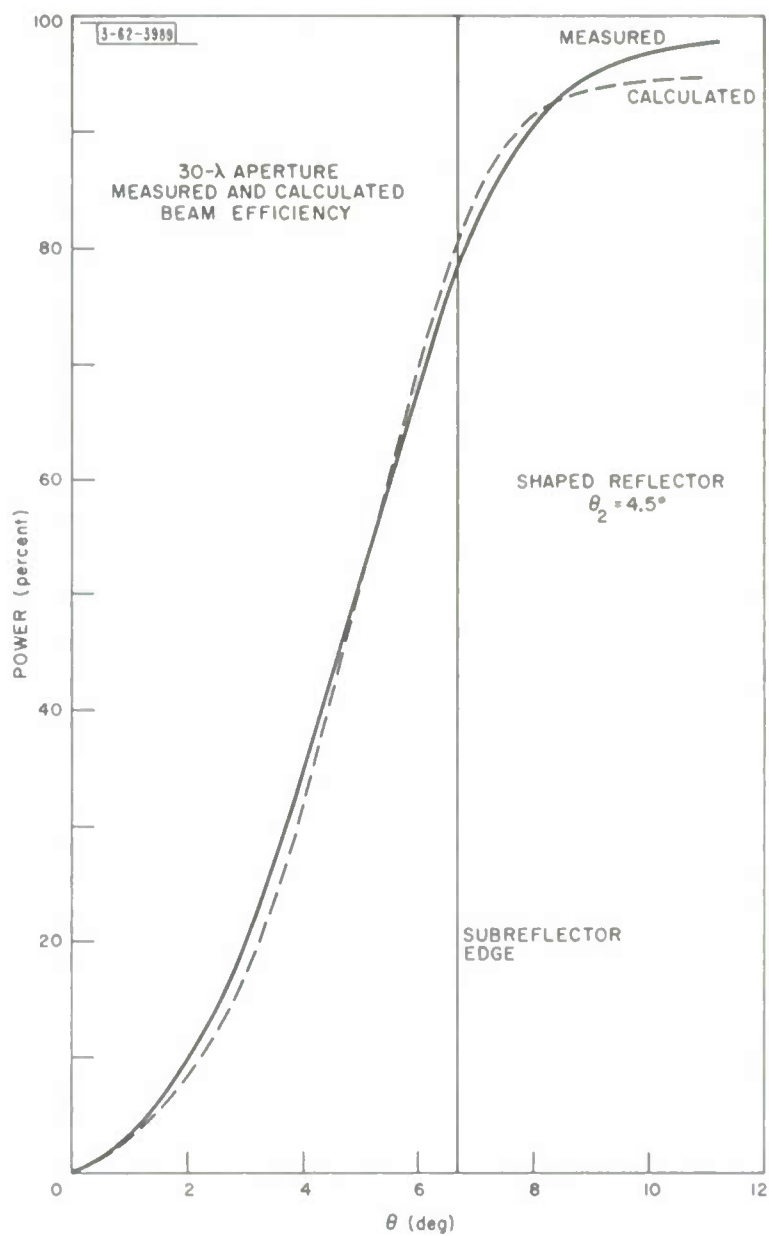
Fig. 11   Comparison of calculated and measured beam efficiency of
shaped reflector ($\theta_2 = 4.5^\circ$)

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Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Lincoln Laboratory, M.I.T.	Unclassified
	2b. GROUP
	None

3. REPORT TITLE

A Shaped Reflector as a Primary Feed for Haystack

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Technical Note

5. AUTHOR(S) *(Last name, first name, initial)*

Sotiropoulos, Arthur

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
30 September 1965	25	7

8a. CONTRACT OR GRANT NO. AF 19(628)-5167	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 649L	Technical Note 1965-15
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	ESD-TDR-65-457

10. AVAILABILITY/LIMITATION NOTICES

None

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Air Force Systems Command, USAF

13. ABSTRACT

This paper describes a technique for obtaining high aperture efficiency in the Haystack 120-foot diameter Cassegrain antenna system. The approach used seeks to maximize the percentage of power intercepted by the hyperboloidal subreflector of the Cassegrain antenna system by shaping its primary feed radiation pattern. The primary feed configuration is a linearly polarized waveguide feed illuminating a specially shaped reflector. A number of theoretical antenna models were calculated and one of these antennas (a 30 wavelength diameter shaped reflector at 7750 Mcps) was fabricated and evaluated. Calculated estimates of percentage of primary feed power intercepted by the subreflector and Cassegrain aperture efficiency are presented.

14. KEY WORDS

antenna feed design
Cassegrainian system
Haystack